Bangladesh Development Studies
Vol. XXXV, December 2012, No. 4

Measuring the Cost of Environment-Friendly Textile Processing in Pakistan: A Distance Function Approach

SAMINA KHALIL*

The effective regulation of industrial water pollution can only be ensured when the regulatory charge/tax is equivalent to the cost of pollution abatement. Pakistan's principle manufacturing sector is textile industry which is termed as the back bone of its economy and earns major share of foreign exchange for the country. The textile processing is the largest water polluting sub-sector of textile industry in Pakistan. The input and output data from 45 textile processing firms are used for the present analysis. Distance function values are applied using ordinary least squares method to estimate parameters for a translog form of the output distance function for textile processing firms in Pakistan. Shadow prices of BOD and COD (undesirable outputs) are obtained from the translog parameter estimates. These prices are negative for all 45 firms in the sample showing consistent results with those found in other studies. These shadow prices are useful estimates which can be used to set pollutant specific emissions charges/taxes corresponding to the existing emission standards so that the polluting firm will have incentives to comply with the standards.

Keywords: Undesirable Output, Shadow Prices, Data Envelopment Analysis, Marginal Abatement Cost, BOD and COD
JEL Classification: Q2, Q5, L6, H2

I. INTRODUCTION

The deliberations in this paper are based on the premise that if the water polluting textile processing industry in Pakistan has to comply with environmental regulations, the demand for the polluted wastewater disposal or pollution control services by the industry should depend upon the cost of pollution abatement. The supply and demand prices for environmental services

*Senior Research Economist, Applied Economics Research Centre, University of Karachi, Karachi, Pakistan. Email: skhalilpk@yahoo.com; samina.khalil@gmail.com

are viewed as marginal damages to the society from pollution and marginal costs of pollution abatement to the industry respectively. In addition to the government's existing regulatory measures, economic instruments like pollution tax can be persuasive for controlling pollution. Producers could rightly consider environmental regulations in terms of economic instruments like taxes for water pollution control as productive inputs for which they have to pay. According to Murty (2000), "Pollution tax is nothing but the price of waste disposal services with respect to which supply is equal to demand." As the environmental quality standards are generally pronounced for a range of pollutants, as is the case of NEQS (National Environmental Quality Standards) in Pakistan, there has to be pollutant specific tax corresponding to given standards. This paper explicitly deals with the output distance function technique that is applied to estimate the marginal or shadow cost of water pollution abatement. Section II contains theoretical description of the output distance function. Details of empirical estimations of shadow prices of water pollutants data for the textile processing units in Pakistan are dealt in section III. Finally conclusion is given in section IV.

II. COST OF POLLUTION ABATEMENT MODEL AND METHODOLOGY

There are several cost models, developed and used in empirical studies, focusing on the estimation of the cost of water and air pollution. Output distance function approach is the most reliable among these models. The cost of pollution abatement includes both point and non-point sources of pollution. This cost has critically important policy implications for firms from a wide range of industries across all sectors of an economy.

2.1 The Output Distance Function

Shephard (1970) came up with the idea of driving shadow prices using output distance function and the duality results. The definition of the maximum output in conventional production function is one that can be produced from an exogenously given input vector, while the cost function defines the minimum cost to produce the exogenously given output. Output distance function and input distance function generalise these notions to a multi-output case.

The empirical application of the output distance function method gives reliable estimates of the prices of pollutants or undesirable by-products in the production of desirable outputs, using plant level data on input and output quantities. The parametric form of the output distance function is estimated using econometrics. The duality theorem between the output distance and revenue functions is applied to derive shadow prices of pollutants.

In this paper, output distance function is used to derive shadow prices of BOD5 (Biological Oxygen Demand) and COD (Chemical Oxygen Demand). These estimates are viewed as pollutant specific marginal abatement costs for textile processing units at the actual mix of outputs (Coggins and Swinton 1996, Murty, Surender and Mahua 2006). It has been pointed out that pollution abatement measures adopted by the polluting units reduce a vector of pollutants which subsequently results in joint costs to the abatement of all pollutants and an attributable cost to each pollutant. In Murty's words, "The pollution abatement cost function estimated using one of the pollutants as a proxy for all pollutants while accounting for the cost specific to that pollutant may not be accounting for all the joint cost of pollution abatement." If a tax is designed on the basis of such estimates of the pollution abatement cost function, it will not reflect the appropriate tax rate and consequently provide incorrect signals to the polluting units. As a result, the probability of noncompliance to the desired standards of pollution on the part of producing units is quite high.

Some parametric frontier analysis has attempted to solve the multiple output problems by estimating the production technology using either: (a) an input requirements function (Gathon and Perelman 1992) in which a single (possibly aggregate) input is expressed as a function of a number of outputs; or (b) an output-oriented distance function (Lovell, Richardson, Travers and Wood 1994, Grosskopf, Hayes, Taylor and Weber 1997) which can accommodate both multiple inputs and multiple outputs.[1]

III. EXPLANATION OF ESTIMATION APPROACH USED FOR PARAMETER ESTIMATES OF OUTPUT DISTANCE FUNCTION

In order to estimate the shadow prices of pollution (bad outputs), the parameters of the output distance function need to be estimated. The translog specification for the output distance function with 3 outputs, 3 inputs and technical change is given in the expression (16- Appendix 1.3) where *x* and *y* are Nx1 and Mx1 vectors, respectively of inputs and outputs. There are three inputs

[1] For detailed theoretical explanation of the Output distance function–good and bad outputs, duality between output distance function and revenue function, estimation of shadow prices for undesirable outputs and translog output distance function, see Appendix 1.

(capital, labour and material input) and three outputs (good output, processed/printed fabric and bad outputs BOD and COD). The main problem with the econometric estimation of distance function is that the dependent variable cannot be observed. The technical efficiency values of all firms using VRS (Variable Returns to Scale) model of DEA (Data Envelopment Analysis) with output orientation have been estimated using DEAFrontier software. The output distance function measures the reciprocal of the largest possible proportional increase in the output vector, the extent of which is indicated by the value of θ. The value of the distance function is thus the inverse of the Farrell output measure of technical efficiency based on DEA method. Therefore by taking the inverse (reciprocal) of the estimated technical efficiency values, the problem of unobserved values of the dependent variable is solved.

3.1 Data and Estimations

The data used in this paper are from primary survey of 45 textile processing firms for the years 2004–2008. It consists of variables such as: Y, output as sales value in million rupees; BOD and COD load in tons; CAPTL, capital stock in million rupees; LABR, labour employed in numbers; MATINP, material input cost in million rupees. The variables used for estimation with monetary values are deflated values with the base year prices in 2004-05. For estimation of output distance function, the technology of each water polluting firm is described by joint outputs: sales value (desirable output) and BOD, COD (undesirable outputs) and inputs: capital, labour and material. Thus the information collected constitutes balanced panel data with 225 observations on all of the relevant variables.

TABLE I

DESCRIPTIVE STATISTICS OF VARIABLES USED IN ESTIMATION

Variable	Mean	Minimum	Maximum	Standard Deviation
Y	2697.281	737.173	6058.043	1053.201
LABR	2570	966	4521	1032.01
MATINP	1750.554	804.32	10976.320	3746.201
CAPTL	2798.076	1580.510	8053.025	982.510
BOD load	338.304	0.026	1511.975	261.083
COD load	1219.402	31.287	9594.437	1087.005

Results from the estimation of translog specification and OLS parameter estimates of the output distance function for the textile processing firms are presented in Appendix 2. The coefficient of determination R^2 is a global fit of the model. The goodness of fit of the model has reasonably high level of variation in the response variable that is technical efficiency which is an output distance function explained by the explanatory variables. Almost 98 per cent of the variation in the technical efficiency scores of the firms or distance function, which is response variable, is being explained by the variables on the right hand side of the function. The estimated t values on almost all first order coefficients and majority of second order coefficients are in excess of 1.96 in absolute value. The sum of the first order input coefficients has value greater than 1 which indicates the increasing returns to scale at the mean and positive marginal productivities of inputs are also indicated. In the estimated output elasticity of each input, the share of Capital (0.6413) with a very significant t value shows that capital has important contribution in net technical efficiency.

The parameter estimates of output distance function were used to derive shadow prices of undesirable outputs: BOD and COD. The shadow price of an undesirable output is defined as the value of desirable output lost by the polluting firm to reduce one unit of desirable output which in this case is printed fabric. The interpretation of shadow price could also be the marginal cost of pollution abatement.

3.2 Derivation of Shadow Prices for Undesirable Outputs

The absolute shadow prices for BOD and COD for each mill, r_2 and r_3, are computed in two stages. The first stage involves the calculations of normalised shadow prices, $r^*(x,y)$, for both desirable and undesirable outputs. These values are needed in the next stage for the final calculations of the absolute shadow prices of undesirable outputs.

Derivation of Shadow Prices for BOD and COD

Normalised Shadow Prices for Printed Fabric (PF- good output)

$$r_1^* (x, y_1,y_2,y_3) = \frac{\partial Do(\mathbf{x}, \mathbf{y})}{\partial y_1} = \alpha_1 \frac{Do(\mathbf{x}, \mathbf{y})}{y_1}$$

where y_1 is PF, y_2 is BOD, y_3 is COD and Do is distance function value

Normalised Shadow Prices for BOD (bad output)

$$r_2^* (x, y_1,y_2,y_3) = \frac{\partial Do(\mathbf{x}, \mathbf{y})}{\partial y_2} = \alpha_2 \frac{Do(\mathbf{x}, \mathbf{y})}{y_2}$$

Normalised Shadow Prices for COD (bad output)

$$r_3^* (x, y_1,y_2,y_3) = \frac{\partial Do(\mathbf{x}, \mathbf{y})}{\partial y_3} = \alpha_3 \frac{Do(\mathbf{x}, \mathbf{y})}{y_3}$$

Assume market price of PF (r_1) = absolute shadow price of PF (r_1^0)

revenue for each firm $R(x,r) = r_1^0 / r_1^* (\mathbf{x}, \mathbf{y_1})$

shadow prices for BOD $r_2 = R(\mathbf{x},\mathbf{r}) \bullet r_2^* (\mathbf{x}, \mathbf{y})$

To find the values for r_i^* (**x, y**), we make use of equation (7- Appendix 1.2)

$$r_1^*(x, y_1, y_2, y_3) = \frac{\partial Do(x, y_1, y_2, y_3)}{\partial y_1}$$

and

$$r_2^*(x, y_1, y_2, y_3) = \frac{\partial Do(x, y_1, y_2, y_3)}{\partial y_2} \text{ and } r_3^*(x, y_1, y_2, y_3) = \frac{\partial Do(x, y_1, y_2, y_3)}{\partial y_3}$$

The parameter, say α_1, from the translog function (16- Appendix 1.3) may be interpreted as a measure of elasticity of the distance function Do(x,y) with respect to y_1, that is, the percentage change in Do (x,y) for a small percentage change in y_1. This elasticity coefficient is defined as:

$$\alpha_1 \frac{Do(x,y)}{\partial_{y_1}} \cdot \frac{y_1}{Do(x,y)}$$

or

$$\frac{\partial Do(\mathbf{x, y})}{\partial y_1} = \alpha_1 \frac{Do(\mathbf{x, y})}{y_1} \qquad (1)$$

As we have the estimated values of α_1, Do(**x, y**) from translog function and taking y_1 from the data sets, we can calculate a value for ∂Do(**x, y**) / ∂y_1 and thus a corresponding value for r_1^* (x,y).

Substituting the estimated parameter values of α_1, α_2, α_3 and Do (x,y) into (1) for firm 1, we obtain:

$$r_1^*(x, y_1, y_2, y_3) = \frac{\partial Do(x, y)}{\partial y_1} = \alpha_1 \frac{Do(\mathbf{x, y})}{y_1} = 0.5305 \; \frac{0.849}{5324} = 0.0000845 \qquad (2)$$

$$r_2^*(x, y_1, y_2, y_3) = \frac{\partial Do(x, y)}{\partial y_2} = \alpha_1 \frac{Do(\mathbf{x, y})}{y_2} = -0.5398 \; \frac{0.849}{1224} = -0.0003744 \qquad (3)$$

The next step is to obtain the absolute shadow prices for BOD, where we may assume that the absolute shadow price for PF (r_1) equals the observed market price of PF ($r_1 = r^0$) where the average annual price for local delivered PF in 2004 is Rs 4,830/tons (MoE, Pakistan 2006).

$$r_1 = r_1^0 = 4830 \qquad (4)$$

Next, we calculate the revenue, R(x,r), for each firm using (4) where we make use of the value of r_1^* (x,y) obtained earlier and the market price of PF, so that:

$$R(x,r) = r_1^0 \; / \; r_1^*(\mathbf{x, y}) = 4830 / 0.0000845 = 57{,}1597{,}63 \text{ for firm 1} \qquad (5)$$

Once we obtained the value of R (x,r) for each firm, we then derive absolute shadow prices for the undesirable output (BOD), r_2, for each firm using (13) with estimated r_2*value from (3) :

$$r_2 = R(x, r).r_2^*(x, y) = (57,1597,63)(-0.0003744) = -21400.615 \quad (6)$$

Alternatively, once the values of r_1* and r_2* are computed, to attain the values of r_2 , we may rewrite (14) as:

$$r_2 = r_1^0 \frac{\partial Do(x, y) / \partial y_2}{\partial Do(x, y) / \partial y_1} = r_1^0 \frac{r_2^*(x,y)}{r_1^*(x,y)} \quad (7)$$

As the values of r_1* and r_2* have already been computed and with the given market price of PF as Rs 4830 per ton, we finally arrived at the value of the absolute shadow price of BOD for firm 1, which is equal to:

$$r_2 = (4830)\frac{(-0.0003744)}{(0.0000845)} = -21400.615 \quad (8)$$

The shadow price of BOD for all firms is negative or less than zero indicating forgone revenue (or PF) if firms try to reduce BOD by 1 unit (ton) per year. The value of Rs - 21400.615 for the shadow price of BOD means that if BOD emissions are reduced by 1 ton per year, there will be less resources available to produce PF ; hence, the production of PF decreases by 4.483 tons per year, given the market price of Rs. 4,830. The variation in production process to produce various forms of printed fabrics and finished product, resulting in different emission levels, is depicted in firm specific shadow prices.

Similarly, the shadow price for COD (r_3) is calculated for firm 1 by computing the value of r_3 * (x, y), given the computed values of r_1* and market price of PF.

$$r_3^*(x, y_1, y_2, y_3) = \frac{\partial Do(x, y)}{\partial y_3} = \alpha_3 \frac{Do(\mathbf{x}, \mathbf{y})}{y_3} = -0.1809 \frac{0.849}{209} = -0.0007348$$

$$r_3 = r_1^0 \frac{r_3^*(x,y)}{r_1^*(x,y)} = (4830)\frac{(-0.0007348)}{(0.0000845)} = -42000.994 \quad (9)$$

TABLE II

DESCRIPTIVE STATISTICS OF OUTPUT DISTANCE FUNCTION VALUES AND SHADOW PRICES OF POLLUTANTS

Pollutant	Mean	Minimum	Maximum	Standard Deviation
D.F	0.90435	0.58045	1.0000	0.7761
BOD	67248.60	20441.30	138320.10	82310.97
COD	125545.0	36421.08	228651.00	127182.52

3.3 Numerical Results of Shadow Price Estimation

The numerical results of shadow price estimation are shadow prices of undesirable outputs (see Appendix 3) based on parameter estimates of output distance function given in Appendix 2.

The shadow prices based on output distance function account for the cost of all pollution abatement methods used by the firms to meet the prescribed standards (Environmental Quality Standards), but the firms face resource constraints such that they have to reduce pollution (BOD and COD) at the cost of reducing the production of good output which is printed fabric, taken in tons from the data set in this case. The firm–specific shadow prices of BOD and COD are less than zero, which reflects good output and revenue forgone as a result of reducing the level of effluent by one unit (ton) per year. The average shadow price of BOD for 45 firms is Rs. 67,248.60. Given the average market price of printed fabric, which is Rs. 4,830, reduction of one ton of BOD takes the resources off from the production of 13.92 tons of printed fabric in a year. Similarly, the estimated mean shadow price of COD is Rs. 125,545, which results in the reduction of 25.99 tons of desirable output per year given the market price of desirable output. The large variation in the values of firm specific shadow prices gives large standard deviation as compared to the mean values in Table II. The large standard deviations in shadow prices of firms can be explained by the heterogeneity in the production processes of these firms, which implies that there is significant difference in the cost (and shadow price) of abatement across various processes adopted for the production of desirable output. Applying the same argument as in Fare, Grosskopf, Lovell, Yaisawarng (1993) regarding variation in shadow prices across firms, it can be said that the existing pollution regulations have not been able to achieve the efficient allocation of resources. They are of the view that the marginal reduction in effluents that gives same environmental benefits to firms should imply that efficient regulation is characterised by equal marginal costs or shadow prices of the reduction in effluents across firms. These relatively uniform benefits are subject to geographical proximity of firms. Although the sample of textile processing firms used in this paper comes from two different geographical locations, equal marginal cost or shadow prices may be applicable for a shared location by firms.

3.4 Marginal Abatement Cost Function

The shadow price of undesirable output is defined as the value of the desirable output lost by the polluting firm to reduce one unit of undesirable output. This shadow price is also interpreted as the marginal cost of pollution abatement (Murty 2000). Hence, the marginal abatement cost function can be estimated for each pollutant or undesirable output, given the estimated values of shadow prices and data on wastewater volumes (W) and pollution levels (BOD & COD) for 45 firms in the sample using simple regression models.

$$\ln BODS = \alpha_0 + \alpha_1 (\ln W) + \alpha_2 (BOD) \qquad (10)$$

$$\ln CODS = \alpha_0 + \alpha_1 (\ln W) + \alpha_2 (COD)$$

where lnBODS and lnCODS are the logs of shadow prices of BOD and COD respectively.

$$\ln BODS = \underset{(9.908)}{10.789} - \underset{(-5.293)}{0.155}\ln W - \underset{(-8.286)}{0.344}BOD \qquad R^2 = 0.41 \qquad (11)$$

$$\ln CODS = \underset{(12.878)}{11.043} + \underset{(5.920)}{0.172}\ln W - \underset{(-9.848)}{0.367}COD \qquad R^2 = 0.45$$

(Figures in brackets are t values)

BODS: BOD shadow price

CODS: COD shadow price

W: Wastewater volume - KL

The estimated marginal abatement cost function is a derived function from the estimated output distance function as the data on the dependent variable are estimates of the firm specific shadow prices of BOD and COD. In expression (10), the variables BOD and COD represent pollution concentration in a litre of water. All coefficients are significant at 5% or 10% level. The signs of both pollutants are negative, implying that there are raising marginal costs of pollution abatement, the lower the pollution concentration, the higher the marginal cost. Also, the estimates show that the shadow price of undesirable outputs falls with the wastewater volume in the case of BOD but positive sign of coefficient of W implies raising marginal cost in the case of COD. The policy implications and the impact of appropriate regulatory measures on the output of textiles depend upon the right values of estimated tax rate (economic instrument) that can be adjusted until it creates the most effective incentive for producer to keep the output at the profitable level.

IV. CONCLUSIONS

The output distance function and its dual, the revenue function has been used in this paper to measure shadow prices of undesirable outputs. These functions are already established in the literature as an alternative technique. The maximum output in terms of conventional production function is defined as the output that can be produced from an exogenously given input vector while the output distance function generalises this notion to a multi-output case including the undesirable outputs. This approach is particularly useful when detailed data on individual firm's abatement costs are not available. The shadow prices can be estimated by using data detailing inputs and outputs of each firm, which could be interpreted as the marginal cost of pollution abatement. The data are used applying distance function values to ordinary least square method to estimate parameters for a translog form of the output distance function for textile processing firms in Pakistan. Shadow prices of BOD and COD (undesirable outputs) are obtained from the translog parameter estimates. These prices are negative for all 45 firms in the sample showing consistent results with those found in other studies. These shadow prices are useful estimates which can be used to estimate pollutant specific emissions taxes corresponding to the existing emission standards so that the polluting firm will have incentives to comply with the standards.

REFERENCES

Coggins. J. S. and J. R. Swinton. 1996. "The Price of Pollution: "A Dual Approach to Valuing SO_2 Allowances." *Journal of Environmental Economics and Management,* 30: 58-72.

Coelli, T. and S. Perelman. 2000. "Technical Efficiency of European Railways: A Distance Function Approach." *Applied Economics*, Taylor and Francis Journals, 32(15): 1967-76.

Färe, R., S. Grosskopf, C. A. K. Lovell and S. Yaisawarng. 1993. "Derivation of Shadow Prices for Undesirable Outputs: A Distance Function Approach." *The Review of Economics and Statistics*, 75: 374–380.

Färe, R. and S. Grosskopf. 1998. "Shadow Pricing of Good and Bad Commodities." *American Journal of Agricultural Economics*, 80 (3): 584-590.

Färe, R. and D. Primont. 1995. *Multi-Output Production and Duality: Theory and Applications.* Boston: Kluwer-Nijhoff Publishing.

Gathon, H. J. and S. Perelman. 1992. "Measuring Technical Efficiency in European Railways: A Panel Data Approach." *Journal of Productivity Analysis,* 3: 135–151.

Grosskolf,S., K. Hayes, L. Taylor and W. Weber. 1997. "Budget Constrained Frontier Measures of Fiscal Equality and Efficiency in Schooling." *Review of Economics and Statistics,* 79: 116–124.

Lovell, C. A. K., S. Richardson, P. Travers and L. L. Wood. 1994. "Resources and Functionings: A New View of Inequality in Australia." In W. Eichhorn (ed.) *Models and Measurement of Welfare and Inequality.* Berlin: Springer-Verlag.

MoE (Ministry of Environment, Pakistan). 2006. *Country Strategy Paper.*

Murty, M N. 2000. "Environmental Regulation and Economics of Environmental Policies." In R.N. Bhattacharya (ed.) *Environmental Economics*. New Delhi: Oxford.

Murty, M. N., Surender K. and P. Mahua. 2006. "Environmental Regulation, Productive Efficiency and Cost of Pollution Abatement: A Case Study of the Sugar Industry in India." *Journal of Environmental Management,* 79.

Murty, M. N. and Surender Kumar. 2002. "Measuring the Cost of Environmentally Sustainable Development in India: A Distance Function Approach." *Environment and Development Economics,* 7(03): 467-486.

Pittman, R.W. 1981. "Issues in Pollution Control Interplant Cost Differences and Economies of Scale." *Land Economics*, 58: 1-17.

Shephard, R. 1970. *Theory of Cost and Production Functions*. Princeton: Princeton University Press.

Swinton, John R. 1998. "At What Cost Do We Reduce Pollution? Shadow Prices of SO2 Emissions." *The Energy Journal,* 19(4): 63-83.

APPENDIX 1

OUTPUT DISTANCE FUNCTION – GOOD AND BAD OUTPUTS

The output distance function is required to be explained in terms of multi-output production in which joint production of a desirable (good) and an undesirable (bad) output situation is observed, which is the focus of this paper. It is imperative to distinguish between good output ($y \in \Re^{M}+$) and bad output ($u \in \Re^{J}+$). In the production context, the former is typically a marketed good and the latter is often not marketed but rather a by product which may have deleterious effects on the environment or human health and therefore its disposal is often subject to regulation. For example, printed fabric is produced from processing grey fabric using chemicals (bleach, dyes). In this case, the desirable marketed output is printed fabric, and among undesirable byproducts, BOD5 and COD are the most significant.

In terms of disposability, we have two alternative assumptions concerning output disposability:

$$(y, u) \in P(x) \Rightarrow 0 \leq \theta \leq 1 \text{ imply } (\theta y, \theta u) \in P(x) \quad (1)$$

$$(y^{o}, u^{o}) \in P(x) \Rightarrow (y, u) \leq (y^{o}, u^{o}), \text{ imply } (y, u) \in P(x) \quad (2)$$

Expression (1) imposes weak disposability of outputs on the technology and depicts the case where feasible output vector $(y, u) \in P(x)$ (where y is a desirable output and u is an undesirable output) is proportionally decreased then it is still feasible that $(\theta y, \theta u) \in P(x)$, where $0 \leq \theta \leq 1$. When desirable and undesirable outputs are jointly produced, and the undesirable output may not be disposed of without cost as there could be some regulatory restrictions, then expression (2) is an appropriate assumption on the technology. Assumption (2) states, if undesirable outputs are decreased then at the margin the desirable outputs should also be decreased, holding inputs x constant. An alternative interpretation is that if we hold inputs constant, then “cleaning up” undesirable outputs will occur at the margin through reallocation of inputs away from the production of desirable outputs.

1.1 Duality between Output Distance Function and Revenue Function

One unique feature of the approach to efficiency measurements is that whether they are difference or ratio based, they are all rooted in duality theory and the dual measures are support functions, such as profit, cost and revenue functions. Primal measures are their dual distance functions. The preference for the use of output distance function is because the duality between the output distance function and the revenue function allows one to retrieve the output shadow prices desired (Färe, Grosskopf, Lovell and Yaisawarng 1993).[2]

---

[2] It is possible to derive shadow prices of inputs by modeling technology with an input distance function, and through its duality to the cost function to seek shadow prices of inputs (Färe and Grosskopf 1990).

The distance function and revenue function is defined as $R(x, r) = \sup_y \{ ry : Do(x, y) \leq 1 \}$. R(x,y) maps out the maximum (or supermum) revenue frontier, given that the firm is constrained by a production possibilities set (Swinton 1998).

The duality of the distance function and the revenue function is given as:

$$R(x, r) = \sup_y \{ ry : Do (x, y) \leq 1 \} \quad (3)$$

$$Do (x, y) = \sup_r \{ ry : R (x, r) \leq 1 \} \quad (4)$$

where r is the revenue and x is inputs

Equality (3) suggests that subject to the constraint $Do(x, y) \leq 1$, maximisation of the revenue (ry) by the producer is achieved by choosing the output mix y given output price vector r. The equation (4) implies that given the constraint $R(x,r) \leq 1$, the producer picks the optimal output prices that maximise revenue (ry) given y. It restricts on the revenue curve to the frontier or below it.

$$r = R(x, r) \frac{\partial Do(x, y)}{\partial y} y \quad (5)$$

Expression (5) shows that the output shadow prices r equal the gradient of the output distance function times maximal revenue (Färe and Grosskopf 1998).

1.2 Estimation of Shadow Prices for Undesirable Outputs

The shadow prices can be obtained by selecting the optimal vector of prices, given the gradient vector, $(\partial Do(x, y) / \partial y)$, such that total revenue R(x, r) is maximised. This is done by using another part of the duality theorem (3), which can be written as:

$$Do(x,y) = r^* (x,y)y \quad (6)$$

where r(x,u) is given by second duality (4) which is the revenue maximising output price vector where the total revenue ry is maximised with respect to output prices.

Shepherd's dual lemma is applied to (6) to get:

$$\frac{\partial Do(x, y)}{\partial y} = r^* (x, y) \quad (7)$$

Substituting (7) into (5) gives:

$$r = R(x, r)\, r^* (x, y) \quad (8)$$

$$r^* (x, y) = \frac{r}{R(x,r)} \quad (9)$$

Vector $r^* (x, y)$, which is derived from Shephard's dual lemma, is the normalised or revenue deflated output shadow prices and r is the un-deflated shadow prices (Färe and Primont 1995, Fare and Groskopf 1998).

Färe and Primont (1995) and Fare and Groskopf (1998) employ the following assumption in their analyses: on observed output price equals its absolute shadow price and it is quite appropriate to assume that maximum revenue is equal to its observed revenue. Using the assumption and denoting the observed market price of good or desirable output by r_g and its revenue deflated shadow price by r_g * allows calculation of maximum revenue as follows:

$$R\,(x,r) = r_g{}^0 / r_1 * \,(x,y) \tag{10}$$

For all other outputs, i ≠ 1, absolute shadow prices ri are given as:

$$r_i = R(x, r)\, r_i^* (x, y) \tag{11}$$

Considering (10) and (11) together produces

$$r_i = R(x, r)\, r_i^* (x, y) = [\, r_g / r_g * \,(x, y)\, r_i^* (x, y)\,] \tag{12}$$

Alternatively, using (7), the expression (12) can be written as:

$$r_i = R(x, r).\, r_i^* (x, y) = R\,(x, r).\, [\partial Do(x, y) / \partial y\,] \tag{13}$$

Now the combination of (7), (10), and (13) gives the expression which is used to compute the shadow price of an undesirable output, i ≠ 1:

$$r_i = \frac{r_g}{r_g * \,(x, y)} \,[\partial Do(x, y) / \partial yi] = r_g \,.\, \frac{\partial Do(x, y) / \partial y_i}{\partial Do(x, y) / \partial y_1} \tag{14}$$

Rearranging (8) gives a familiar expression where relative price equals the corresponding ratio of distance function derivatives:

$$\frac{r_i}{r_g} = \frac{\partial Do(x, y) / \partial y_i}{\partial Do(x, y) / \partial y_1} \tag{15}$$

1.3 Translog Output Distance Function and Data for the Textile Processing Industry in Pakistan

Distance functions have been estimated using a variety of methods in recent years. Data Envelopment Analysis (DEA); Parametric Deterministic Linear Programming (PLP); Corrected Ordinary Least Square (COLS); and Stochastic Frontier Analysis (SFA).

The COLS method has the advantages of being easy to estimate and also permits the conduct of traditional hypothesis test. Furthermore, Collie and Perelman (2000) found that COLS and DEA gave quite consistent technical efficiency rankings when applied to a single data set.

The model specified for this paper includes both desirable and undesirable outputs and several inputs; the translog production function is best suited to estimating such a model (Pittman 1981). Moreover, the data set consists of positive values of input and output quantities, confirming the pertinence of this model for the empirical estimation.

$$\ln D_{oi}(x, y) = \alpha_0 + \sum_{n=1} \beta_n \ln x_{ni} + \sum_{m=1} \alpha_m \ln y_{mi}$$

$$\frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N} \beta_{nn'} (\ln x_{ni})(\ln x_{n'i}) + \frac{1}{2}\sum_{m=1}^{M}\sum_{m'=1}^{M} \alpha_{mm'} (\ln y_{mi})(\ln y_{m'i}) +$$

$$\sum_{n=1}^{N}\sum_{m=1}^{M} \gamma_{nm} (\ln x_{ni})(\ln y_{mi}) \qquad i = 1, 2,\ldots,N \qquad (16)$$

Hence, following Lovell, Richardson, Travers and Wood (1994), one of the outputs can be arbitrarily chosen, such as the Mth output, and set $w = 1 / y_{Mi}$ one obtains,

For the translog form, this provides:

$$\ln (D_{oi} / y_{Mi}) = \alpha_0 + \sum_{n=1}^{N} \beta_n \ln x_{ni} + \sum_{m=1}^{M-1} \alpha_m \ln y^*_{mi} +$$

$$\frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N} \beta_{nn'} (\ln x_{ni})(\ln x_{n'i}) + \frac{1}{2}\sum_{m=1}^{M-1}\sum_{m'=1}^{M-1} \alpha_{mm'} (\ln y_{mi})(\ln y_{m'i}) +$$

$$\sum_{n=1}^{N}\sum_{m=1}^{M-1} \gamma_{nm} (\ln x_{ni})(\ln y^*_{mi}) \qquad i = 1, 2,\ldots,N \qquad (17)$$

where $y^*_{mi} = y_{mi} / y_{mi}$

we can write the following

$$\ln (D_{oi} / y_{mi}) = TL (x_i,y_i) / y_{mi}, \alpha, \beta\gamma) \quad i = 1, 2,\ldots, N \qquad (18)$$

or

$$\ln (D_{oi} - y_{mi}) = TL (x_i,y_i) / y_{mi}, \alpha, \beta\gamma) \quad i = 1, 2,\ldots, N \qquad (19)$$

and hence

$$- \ln (y_{mi}) = TL (x_i,y_i) / y_{mi}, \alpha, \beta\gamma) - \ln (D_{oi}) \quad i = 1, 2,\ldots, N \qquad (20)$$

Given the data, the parameters in (17) of the translog function can be estimated which ensures that the function fits the observed data "as closely as possible," while maintaining the requirement that $0 < D_{oi} \leq 1$, which implies that $-\infty < \ln (D_{oi}) \leq 0$.

Following Lovell, Richardson, Travers and Wood (1994), the corrected ordinary least squares method can be used to estimate an output distance function. The function is fitted in two steps. The first step involves interpreting the unobservable term "$- \ln (D_{oi})$" in (19) as a random error term and estimating the translog distance function using OLS. In the second step, the OLS estimate of the intercept parameter, α_0, is adjusted (by adding the largest negative OLS residual to it) so that the function no longer passes through the centre of the observed points but bounds them from above. The distance measure of the ith firm is then calculated as the exponent of the corrected OLS residual.

APPENDIX 2
PARAMETER ESTIMATES OF OUTPUT DISTANCE FUNCTION

Variable	Parameter	Value	t-value	Variable	Parameter	Value	t- value
Constant	α0	-7.274	- 0.090	X_{23}	β_{23}	0.034	0.082
X_1	β_1	0.6413	9.059	Y_{12}	α12	-5.20E-04	-0.013
X_2	β_2	0.4442	6.078	Y_{13}	α13	0.000303	0.0901
X_3	β_3	0.0971	4.708	Y_{23}	α23	-0.134	-2.443
Y_1	α1	0.5305	8.928	$Y_1 X_1$	γ 11	-0.0406	-7.671
Y_2	α2	-0 .539	-9.122	$Y_1 X_2$	γ 12	0.0131	2,107
Y_3	α3	-0.1809	-5.773	$Y_1 X_3$	γ 13	-0.0024	-1.004
X_{11}	β_{11}	0.0014	2.599	$Y_2 X_1$	γ 21	-0.0368	-6.247
X_{22}	β_{22}	-0.267	-1.454	$Y_2 X_2$	γ 22	0.04472	7.718
X_{33}	β_{33}	0.524	2.171	$Y_3 X_3$	γ 23	-0.0097	-3.708
Y_{11}	α11	- .31E-05	-0.671	$Y_3 X_1$	γ 31	0.0726	1.875
Y_{22}	α22	0.001	0.379	$Y_3 X_2$	γ 32	-0.04479	-3.708
Y_{33}	α33	0.012	0.082		γ 33	0.000738	0.358
X_{12}	β_{12}	3.28E-04	1.65				
X_{13}	β_{13}	-0.078	-0.572		R^2	0.987	

X_1 - Capital Y_1 - Output (Desirable)
X_2 - Material Input Y_2 - BOD (Undesirable output)
X_3 - Labour Y_3 - COD (Undesirable output)

APPENDIX 3

DISTANCE FUNCTION VALUES AND SHADOW PRICES OF BOD AND COD

Firms	Distance Function values	Shadow Prices of BOD	Shadow Prices of COD
1	0.849	-21400.61	-42000.00
2	0.990	-29603.43	-62992.90
3	0.914	-31625.02	-82903.35
4	1.000	-20702.55	-153891.78
5	1.000	-38321.02	-99872.08
6	0.953	-65223.46	-192643.49
7	0.841	-89213.09	-216390.27
8	0.810	-51872.21	-87362.68
9	1.000	-20441.30	-174900.34
10	1.000	-67432.88	-163987.99
11	1.000	-48712.73	-219073.87
12	0.752	-102611.52	-208751.90
13	1.000	-84172.47	-183520.55
14	1.000	-72218.20	-73659.47
15	0.932	-60426.33	-192548.20
16	0.721	-120420.31	-218265.99
17	0.890	-97225.65	-186376.63
18	1.000	-37215.27	-36421.08
19	1.000	-39524.65	-83763.27
20	1.000	-57643.40	-69376.56
21	1.000	-118640.31	-183909.75
22	0.872	-62982.65	-219287.65
23	0.940	-50233.02	-94673.94
24	0.916	-28615.40	-227640.50
25	1.000	-42865.69	-95620.68
26	0.812	-72910.81	-165439.57
27	0.803	-92016.52	-193547.40
28	0.580	-124013.77	-218732.59
29	0.688	-138320.0	-184897.47
30	1.000	-61837.88	-48790.99
31	0.702	-82299.37	-164986.07
32	1.000	-38420.40	-59734.83
33	0.990	-58352.12	-174864.20
34	1.000	-75231.29	-193765.68
35	0.861	-63209.74	-217456.35
36	0.833	-120739.60	-223568.60
37	0.706	-93520.12	-228651.00
38	0.717	-119352.83	-189406.30
39	0.835	-38254.11	-174498.98
40	1.000	-55034.10	-99047.47
41	1.000	-62431.90	-148970.39
42	0.928	-49830.43	-97899.80
43	1.000	-63082.32	-164879.64
44	0.861	-88735.44	-154894.39
45	1.000	-69251.04	-188990.50